

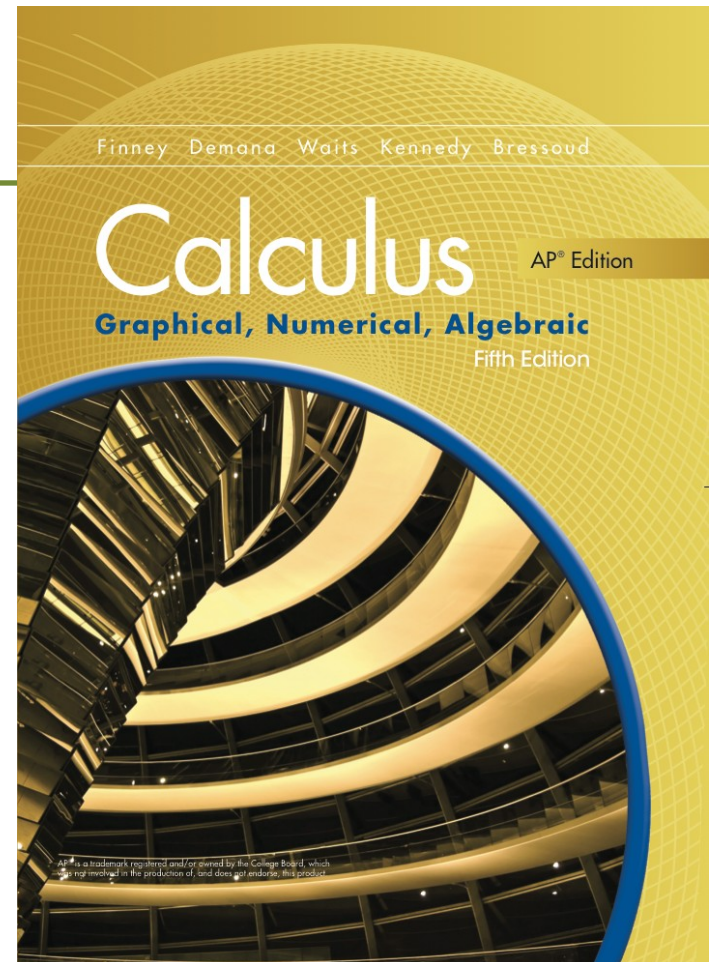
Chapter 6

Applications of Derivatives



Section 6.4

Fundamental Theorem of Calculus



Quick Review

Find dy/dx

1. $y = \sin x^3$

2. $y = (\sin x)^3$

3. $y = \ln 3 - \ln 7$

4. $y = \sin^2 x + \cos^2 x$

5. $y = 3^x$

6. $y = \frac{x}{\cos x}$

7. $y = \sin t$ and $x = 2t$

8. $dx/dy = 2x$

Quick Review Solutions

Find dy/dx

1. $y = \sin x^3$ $dy/dx = 3x^2 \cos x^3$

2. $y = (\sin x)^3$ $dy/dx = 3(\sin x)^2 \cos x$

3. $y = \ln 3 - \ln 7$ $dy/dx = 0$

4. $y = \sin^2 x + \cos^2 x$ $dy/dx = 0$

5. $y = 3^x$ $dy/dx = 3^x \ln 3$

6. $y = \frac{x}{\cos x}$ $dy/dx = \frac{\cos x + x \sin x}{\cos^2 x}$

7. $y = \sin t$ and $x = 2t$ $dy/dx = \frac{\cos t}{2}$

8. $dx/dy = 2x$ $dy/dx = \frac{1}{2x}$

What you'll learn about

- The Antiderivative Part of the Fundamental Theorem of Calculus
- Use of definite integrals to define new functions (accumulator functions)
- The Evaluation Part of the Fundamental Theorem of Calculus
- Evaluation of definite integrals using antiderivatives

... and why

The Fundamental Theorem of Calculus is a Triumph Of Mathematical Discovery and the key to solving many problems.

The Fundamental Theorem of Calculus

If f is continuous on $[a, b]$, then the function

$$F(x) = \int_a^x f(t) dt$$

has a derivative at every point x in $[a, b]$, and

$$\frac{dF}{dx} = \frac{d}{dx} \int_a^x f(t) dt = f(x).$$

The Fundamental Theorem of Calculus

$$\frac{d}{dx} \int_a^x f(t) dt = f(x)$$

Every continuous function f is the derivative of some other function.

Every continuous function has an antiderivative.

The processes of integration and differentiation are inverses of one another.

Example Applying the Fundamental Theorem

Find $\frac{d}{dx} \int_a^x \sin t dt$.

$$\frac{d}{dx} \int_a^x \sin t dt = \sin x$$

Example The Fundamental Theorem with the Chain Rule

Find dy/dx if $y = \int_1^{x^2} \sin t dt$.

$$y = \int_1^{x^2} \sin t dt$$

$$y = \int_1^u \sin t dt \quad \text{and } u = x^2.$$

$$\text{Apply the chain rule: } \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \left(\frac{d}{du} \int_1^u \sin t dt \right) \cdot \frac{du}{dx}$$

$$= \sin u \cdot \frac{du}{dx}$$

$$= \sin u \cdot 2x$$

$$= 2x \sin x^2$$

Example Variable Lower Limits of Integration

Find $\frac{dy}{dx}$ if $y = \int_x^5 t \sin t dt$.

$$\begin{aligned}\frac{d}{dx} \int_x^5 t \sin t dt &= \frac{d}{dx} \left(- \int_5^x t \sin t dt \right) \\ &= - \frac{d}{dx} \left(\int_5^x t \sin t dt \right) \\ &= - x \sin x\end{aligned}$$

The Fundamental Theorem of Calculus, Part 2

If f is continuous at every point of $[a, b]$, and if F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x) dx = F(b) - F(a).$$

This part of the Fundamental Theorem is also called the **Integral Evaluation Theorem**.

The Fundamental Theorem of Calculus, Part 2

$$\int_a^b f(x) dx = F(b) - F(a)$$

Any definite integral of any continuous function f can be calculated without taking limits, without calculating Riemann sums, and often without effort — so long as an antiderivative of f can be found.

Example Evaluating an Integral

Evaluate $\int_1^3 (3x^2 + 1) dx$ using an antiderivative.

$$\begin{aligned}\int_1^3 (3x^2 + 1) dx &= \left[x^3 + x \right]_{-1}^3 \\ &= (3^3 + 3) - ((-1)^3 + (-1)) \\ &= 32\end{aligned}$$

How to Find Total Area Analytically

To find the area between the graph of $y = f(x)$ and the x -axis over the interval $[a, b]$ analytically,

1. partition $[a, b]$ with the zeros of f ,
2. integrate f over each subinterval,
3. add the absolute values of the integrals.

How to Find Total Area Numerically

To find the area between the graph of $y = f(x)$ and the x -axis over the interval $[a, b]$ numerically, evaluate

$$\text{NINT}(|f(x)|, x, a, b)$$